Prediction of Three-Dimensional Hypersonic Flows Using a Parabolized Navier-Stokes Scheme

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A new three-dimensional perfect-gas parabolized Navier-Stokes scheme has been developed to study viscous hypersonic flowfields around realistic re-entry configurations under moderate angle-of-attack conditions. The proposed scheme is unconditionally time-like in the subsonic and supersonic flow regions. It is shown that treating the equation of state in its original algebraic form, rather than its differentiated form, leads to an unconditionally time-like problem. Furthermore, a second-order-accurate smoothing and a pseudounsteady approach is used to improve dramatically the efficiency and computing speed of the scheme without compromising the solution accuracy. The flow around a 7 deg sphere-cone vehicle at 5 deg angle of attack is considered, and the predicted results show that the present scheme is not only accurate, but it is also computationally fast and efficient.

Nomenclature

e = specific total energy
h_0 = total specific enthalpy
J = determinant of the transformation Jacobian
k = thermal conductivity
M = Mach number
$PINF$ = freestream static pressure, p_∞
Pr = Prandtl number
p = static pressure
QW = wall heat-transfer rate, Btu/ft^2/s
Re = Reynolds number, $(\rho V Rn)/\mu$
RN, Rn = nose radius
$RSHK$ = radial location of bow shock
T = static temperature
t = time
u = x component of velocity
u_i = u, v, and w for $i = 1, 2$, and 3
V = total velocity
v = y component of velocity
w = z component of velocity
X,x = coordinate along body axis
x_i = x, y, and z for $i = 1, 2$, and 3
x,y,z = Cartesian coordinates
α = angle of attack, deg
ε = M_∞/Re_∞
θ = perturbation parameter
φ = circumferential angle measured from the windward pitch plane
ξ_i = general curvilinear coordinates
ρ = static density
τ = average computing time per grid point

Superscripts

j = index in ξ_1 direction
n = index for iteration

Subscripts

ℓ = index in ξ_2 direction
w = wall quantity
∞ = freestream quantity
, = partial derivative

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Introduction

OVER the past several years a number of parabolized Navier-Stokes (PNS) schemes have been developed to study the problem of hypersonic re-entry flows. These PNS schemes can be broadly classified as 1) noniterative PNS schemes or 2) iterative PNS schemes. Examples of noniterative PNS schemes are the scheme of Schiff and Steger,[1] the AFWAL PNS[2,4] scheme, and the scheme of Vigneron et al.[3] Examples of iterative PNS schemes are the schemes of Lubard and Helliwell,[5] Helliwell et al.,[6] and Lin and Rubin.[9] Although the schemes of Refs. 1–6 solve the steady-state equations, the scheme of Lin and Rubin[9] solves the unsteady equations by integrating them in the time direction until a steady state is reached. All of the aforementioned PNS schemes (Refs. 1–6 and 9) are conditionally time-like (hyperbolic/parabolic) in the streamwise marching direction. In the available literature, there are two ways in which the problem of departure (streamwise stability) has been analyzed, i.e, either from the point of view of the governing differential equations[1–4] or from the point of view of the differenced form of these governing differential equations.[5,6,9]

The works of Schiff and Steger[1] and Vigneron et al.[3] analyze the problem of departure (streamwise stability) by considering the governing differential equations. Their analyses[1,3] indicated that the eigenvalues of the differential system become complex in the subsonic sublayer region, indicating that the governing equations become elliptic in the subsonic sublayer region, and a marching-like solution scheme becomes ill-posed. On the other hand, Lubard and Helliwell[5] and Helliwell et al.[6] looked at the problem of departure (streamwise stability) by considering the differenced form of the governing equations. They used a Fourier-type analysis and found that the eigenvalues of the amplification matrix had magnitudes greater than 1 in the subsonic region, unless the marching step size was kept greater than a certain minimum value. The work of Rubin and Lin[8] and Lin and Rubin[9] also confirm that a minimum step-size constraint is necessary to prevent departure.

Recently, we have successfully developed and used a completely new (fully iterative) PNS approach that is fast and efficient, and is inherently time-like in the subsonic and supersonic flow regions. The robustness and stability of this PNS scheme is clearly demonstrated by the fact that we have successfully used this PNS scheme to study axisymmetric[10,11] and three-dimensional[12] hypersonic re-entry flows with perfect-gas,[10] finite-rate, and equilibrium chemically reacting[12] air models. This paper presents the mathematical development of this new PNS scheme for three-dimensional perfect-gas flows.

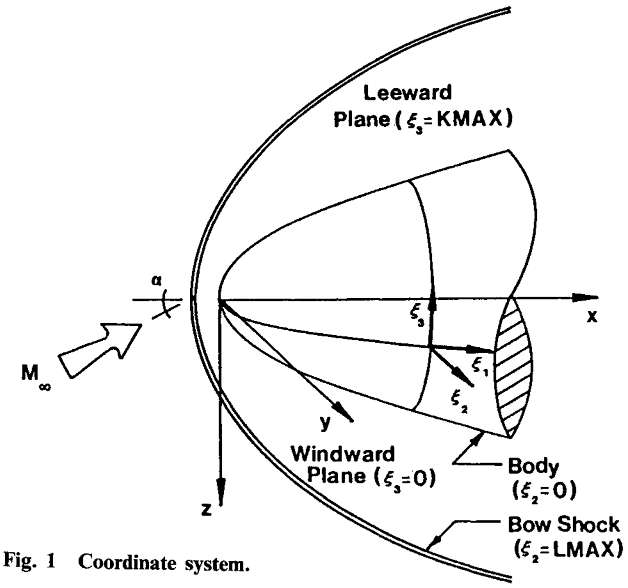

Fig. 1 Coordinate system.

Furthermore, a model differential/algebraic problem is developed and analyzed to study the gross characteristics of this new PNS scheme.

Mathematical Formulation

Coordinate System

The coordinate system used is a general curvilinear coordinate system (ξ_1,ξ_2,ξ_3) shown in Fig. 1. Also, a body-fixed orthogonal (Cartesian) coordinate system is chosen such that the origin of the Cartesian coordinate system is at the tip of the blunt nose, and the x axis is aligned with the axis of the body. The z axis is chosen such that the windward surface of the vehicle is on the positive z-axis side (see Fig. 1). The ξ_1 coordinate is along the body and is also the marching direction. The ξ_2 coordinate is axis-normal and is measured from the body to the outer bow shock. The ξ_3 coordinate is along the circumferential direction and is measured from the windward to the leeward pitch plane. It is assumed that the x,y,z space is uniquely transformable to the ξ_1,ξ_2,ξ_3 space. Thus, at each marching step, every grid cell in the x, y, z space between the j and the $j+1$ step is transformed into a unit cube in the ξ_1,ξ_2,ξ_3 plane with $\Delta\xi_1 = \Delta\xi_2 = \Delta\xi_3 = 1$.

Thin-Layer Parabolized Navier-Stokes Equations

The full Navier-Stokes equations governing three-dimensional compressible flows can be written in a nondimensional form[13] as

$$e_{i,x_i} = \varepsilon g_{i,x_i} + p \tag{1}$$

where the various components of the inviscid vectors e_i and viscous vectors g_i have been derived in Ref. 13. These Navier-Stokes equations have been closed by using the algebraic equation of state for a perfect gas ($\gamma p - \rho T = 0$), which is the last (sixth) equation of the above vectorial set. Unlike the conventional PNS approaches, we choose our unknowns to be the density ρ, the density-velocity products ρu, ρv, and ρw, the density-temperature product ρT, and the pressure p. Thus, our vector of unknowns is

$$q = [\rho,\rho u,\rho v,\rho w,\rho T,p]^T \tag{2}$$

Following the approach of Peyret and Viviand[14] and Viviand,[15] it can be shown that Eq. (1) can be transformed into the following vectorial equation in the general curvilinear coordinate system ξ_i:

$$(f_i - \varepsilon s_i)_{,\xi_i} = (1/J)p \tag{3}$$

$$(f_i - \varepsilon s_i) = (1/J)\xi_{i,x_j}[e_j - \varepsilon g_j] \tag{4}$$

Using a "thin-layer" approximation[1,2] we can neglect the crossflow diffusion effects in favor of the streamwise and axis-normal diffusion effects ($s_{3,\xi_3} = 0$). Thus, if we define $h = (1/J)p$, we obtain the three-dimensional thin-layer Navier-Stokes equations in a general curvilinear coordinate system as

$$f_{i,\xi_i} = \varepsilon(s_{1,\xi_1} + s_{2,\xi_2}) + h \tag{5}$$

Equation (5) is elliptic in both ξ_1 and ξ_2 directions. If we neglect the viscous diffusion and dissipation effects in the ξ_1 direction and assume that the solution can be marched in the ξ_1 direction (the validity of this marching assumption will be discussed later), Eq. (5) reduces to the thin-layer PNS equations. The complete set of these PNS equations can be written in the following vectorial form:

$$\partial\begin{bmatrix} \rho U_i/J \\ (\rho u U_i + \xi_{i,x}p)/J \\ (\rho v U_i + \xi_{i,y}p)/J \\ (\rho w U_i + \xi_{i,z}p)/J \\ \rho h_0 U_i/J \\ 0 \end{bmatrix} / \partial\xi_i = \varepsilon(\partial\begin{bmatrix} 0 \\ s_2 \\ s_3 \\ s_4 \\ s_5 \\ 0 \end{bmatrix}/\partial\xi_2) + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ (\gamma p - \rho T)/J \end{bmatrix} \tag{6a}$$

or

$$f_{i,\xi_i} = \varepsilon s_{,\xi_2} + h \tag{6b}$$

where $U_i = \xi_{i,x_j}u_j$ are the contravariant velocity components. With $m_{jk} = \xi_{2,x_j}\xi_{2,x_k}$, the various elements of the viscous vector s can be written as

$$s_2 = (\mu/J)(m_{kk}u_{,\xi_2} + m_{1j}u_{j,\xi_2}/3) \tag{7a}$$

$$s_3 = (\mu/J)(m_{kk}v_{,\xi_2} + m_{2j}u_{j,\xi_2}/3) \tag{7b}$$

$$s_4 = (\mu/J)(m_{kk}w_{,\xi_2} + m_{3j}u_{j,\xi_2}/3) \tag{7c}$$

$$s_5 = (\mu/J)[m_{kk}(kT_{,\xi_2}/\mu + u_ju_{j,\xi_2}) + m_{jk}u_ju_{k,\xi_2}/3] \tag{7d}$$

Model Differential/Algebraic Marching Problem

The system of equations represented by Eqs. (6) is not a pure differential system, since it consists of five partial differential equations coupled through a sixth equation that is a purely algebraic relation. In the present treatment, this set of governing equations is referred to as a "differential/algebraic system." An important viewpoint to be presented in this section is that the character classification of a differential/algebraic system is significantly different from the classical character classification of purely differential systems. In the case of a differential/algebraic system, the overall character of the system may depend on the way in which the problem is formulated. This idea is new and can be analytically demonstrated on a model differential/algebraic system.

Consider the following system involving three unknowns, φ_1, φ_2, and φ_3:

$$\varphi_{1,x} - \varphi_{2,y} = 0 \tag{8a}$$

$$\varphi_{2,x} - \varphi_{1,y} + 2\varphi_{2,y} + \varphi_{3,y} = 0 \tag{8b}$$

$$a^2\varphi_1 - \varphi_3 = 0 \tag{8c}$$

with initial/boundary conditions

$$\varphi_3(0,y) = a^2\varphi_1(0,y), \qquad \varphi_3(x,0) = a^2\varphi_1(x,0),$$

$$\varphi_3(x,1) = a^2\varphi_1(x,1), \qquad \varphi_2(0,y) = y,$$

$$\varphi_{2,y}(x,0) = 1, \qquad \varphi_2(x,1) = 1 - (a^2+1)x,$$

$$\varphi_1(0,y) = y, \qquad \varphi_{1,y}(x,0) = 1, \qquad \varphi_1(x,1) = 1 + x \tag{9}$$

The solution to Eqs. (8) for these boundary conditions is

$$\varphi_1(x,y) = y + x \tag{10a}$$

$$\varphi_2(x,y) = y - (a^2+1)x \tag{10b}$$

$$\varphi_3(x,y) = a^2(x+y) \tag{10c}$$

This model problem closely resembles the inviscid limit of the governing PNS equations. It involves only first-order derivatives in the two spatial coordinate directions to simulate the convective derivatives of the inviscid limit of the PNS equations. The third equation of this model problem is an algebraic relation and is used to simulate the role of the algebraic equation of state in the PNS equations.

Formulation I

In this approach we can substitute Eq. (8c) into Eq. (8b) and obtain

$$\varphi_{1,x} - \varphi_{2,y} = 0 \tag{11a}$$

$$\varphi_{2,x} + (a^2-1)\varphi_{1,y} + 2\varphi_{2,y} = 0 \tag{11b}$$

or

$$\boldsymbol{\varphi}_x + \boldsymbol{A}\cdot\boldsymbol{\varphi}_y = 0 \tag{11c}$$

where $\boldsymbol{\varphi} = [\varphi_1, \varphi_2]^T$, and the eigenvalues of matrix $\boldsymbol{A}$ are $\lambda_{1,2} = [1+(2-a^2)^{0.5}, 1-(2-a^2)^{0.5}]$. Defining $\varphi_1 = \psi_1 + \psi_2$ and $\varphi_2 = -(\lambda_1\psi_1 + \lambda_2\psi_2)$, we can write Eqs. (11) as

$$\psi_{1,x} + \lambda_1\psi_{1,y} = 0 \tag{12a}$$

$$\psi_{2,x} + \lambda_2\psi_{2,y} = 0 \tag{12b}$$

Thus, we see that Eqs. (12) and, equivalently, Eqs. (11) are time-like if λ_1 and λ_2 are real ($a^2 \le 2$). When λ_1 and λ_2 are not real ($a^2 > 2$), Eqs. (12) and (11) are elliptic in nature. In other words, a marching-like solution of Eqs. (11) will be valid only if $a^2 \le 2$. Furthermore, it can be shown that for $a^2 \le 2$, the analytic solution to Eqs. (11) can be found, and it is the same as given by Eqs. (10).

Formulation II

From the earlier discussion on formulation I of the model problem, we see that the variable a of the model problem is like the speed of sound in the governing PNS equations. Now, for this model problem, if we can devise another formulation such that the variable a no longer contributes to the eigenvalues of the system, it may provide us with a key to attempt a similar treatment of the governing PNS equations.

In formulation I, the actual differential/algebraic system was reduced to a pure differential system. In formulation II, we will attempt to solve directly the actual differential/algebraic system [Eqs. (8)] and look at the character of the resulting system. It is possible to do such an analysis if one looks at Eqs. (8) as the limiting case of a small-perturbation problem. Such an approach is valid as long as the small-perturbation problem being considered allows us to take this limit without any singular behavior. For this purpose, consider the following problem (where $\theta \ge 0$):

$$\varphi_{1,x} - \varphi_{2,y} = 0 \tag{13a}$$

$$\varphi_{2,x} - \varphi_{1,y} + 2\varphi_{2,y} + \varphi_{3,y} = 0 \tag{13b}$$

$$\theta\varphi_{3,x} = a^2\varphi_1 - \varphi_3 \tag{13c}$$

or

$$\boldsymbol{\varphi}_x + \mathbf{A}\cdot\boldsymbol{\varphi}_y = (1/\theta)\mathbf{B}\cdot\boldsymbol{\varphi} \tag{13d}$$

with

$$\varphi_1(0,y) = y, \qquad \varphi_{1,y}(x,0) = 1,$$

$$\varphi_2(0,y) = y, \qquad \varphi_{2,y}(x,0) = 1,$$

$$\varphi_3(0,y) = a^2y, \qquad \theta\varphi_{3,x}(x,0) = a^2\varphi_1(x,0) - \varphi_3(x,0),$$

$$\varphi_1(x,1) = 1 + x$$

$$\varphi_2(x,1) = 1 - (1+a^2)x$$

$$\theta\varphi_{3,x}(x,1) = a^2\varphi_1(x,1) - \varphi_1(x,1) - \varphi_3(x,1) \tag{14}$$

and the eigenvalues of A are $\lambda_{1,2,3} = [2.4142, -0.4142, 0]$. It can be shown that we can write Eqs. (13) as

$$\psi_{1,x} + \lambda_1\psi_{1,y} - \lambda_2(f/\theta) = 0 \tag{15a}$$

$$\psi_{2,x} + \lambda_2\psi_{2,y} + \lambda_1(f/\theta) = 0 \tag{15b}$$

$$\psi_{3,x} - (\lambda_1 - \lambda_2)(f/\theta) = 0 \tag{15c}$$

where $f = [a^2(\psi_1+\psi_2) + (a^2-1)\psi_3]/(\lambda_1-\lambda_2)$, $\varphi_1 = \psi_1 + \psi_2 + \psi_3$, $\varphi_2 = -\lambda_1\psi_1 - \lambda_2\psi_2$, and $\varphi_3 = \psi_3$. Since λ_1 and λ_2 are real, we can see that Eqs. (15) are unconditionally time-like, and a marching-type numerical solution of Eqs. (15) will be unconditionally valid.

In order to answer the question of how the small perturbation problem of Eqs. (13) relates to the original problem of Eqs. (8), we can see that under the limiting condition $\theta \to 0^+$, Eqs. (13) reduce to Eqs. (8), and the initial/boundary conditions given by Eqs. (14) reduce to the actual conditions given by Eqs. (9). Nevertheless, another question also arises: Is it valid to take the limit of Eqs. (15) or, in other words, do Eqs. (15) behave singularly because of the $1/\theta$ factor? To answer this question, the third equation of this system [Eqs. (15)] shows $f/\theta = \psi_{3,x}/(\lambda_1 - \lambda_2)$ for all θ. In other words, f/θ is always defined if $\psi_{3,x}$ is defined for $\theta \ge 0$. The demonstration that $\psi_{3,x}$ is bounded for $\theta \ge 0$ comes from the actual analytic solution of Eqs. (15). One can verify that the analytic solution of Eqs. (15) for $\theta \ge 0$ is

$$\varphi_1(x,y) = y + x \tag{16a}$$

$$\varphi_2(x,y) = y - (a^2+1)x \tag{16b}$$

$$\varphi_3(x,y) = a^2(y+x) - a^2\theta[1-\exp(-x/\theta)] \to a^2(y+x) \tag{16c}$$

It should be noted that we are marching in the x direction, so that our x is always positive and increasing. Thus, we see that with $\theta \to 0^+$, f/θ is not singular even for $\theta = 0$. Furthermore, we see that with $\theta \to 0^+$, the solution to our hypothetical small-perturbation problem approaches the solution to our actual model problem.

In a related study,[16] we have also obtained numerical solutions of the aforementioned model problem. Two different numerical methods were used to either control or eliminate the generation of numerical truncation errors. An explicit Lax method was studied with a Fourier analysis to develop a maximum marching step-size constraint to limit the rate of growth of errors. Also, a bidiagonal implicit scheme was used for zero and nonzero values of θ, along with matrix manipulations such that numerical errors associated with floating-point operations were either eliminated or reduced to insignificant levels. The results of this numerical study confirm our analysis of the model problem. Furthermore, the results show that the solution of the model problem can be properly and accurately marched, provided one controls or eliminates the numerical errors and maintains solution accuracy. Another important idea established by this numerical study was that for marching problems that do not have an asymptotic limit, it is important to control the growth and generation of truncation errors. These truncation errors not only convect and grow in the same way as the actual solution but, in addition, new truncation errors are also generated at each marching step. Thus, after a sufficient number of marching steps, these errors may become large enough to degenerate the solution substantially. In other words, in case of an inaccurate numerical algorithm, the smaller the marching step size becomes, the larger will be the number of times we cause the numerical errors to generate. This behavior could incorrectly be thought of as a problem caused by the small step size, whereas, the small marching step has nothing to do with the problem, except that by doing so the numerical inaccuracies start affecting the solution much sooner. The real solution to these problems lies not in the step-size manipulation, but in improving one's solution accuracy and limiting or eliminating associated numerical truncation errors.

This mathematical exercise clearly suggests the conclusion: There exists a class of "differential/algebraic" system of equations where it is possible to have a conditionally time-like behavior if one formulates the problem in one way, and it is also possible to have the same problem as unconditionally time-like if one formulates the problem in a slightly different manner.

The model problem [Eqs. (8)] presented herein closely resembles the inviscid limit of the governing PNS equations [Eqs. (6)]. The classical treatments of these PNS equations[1-9] correspond to the formulation I presented earlier, which was conditionally time-like. The present scheme, however, follows the approach of formulation II, which has an unconditionally time-like character.

Numerical Formulation

Expansion around the Previous Iteration

In the present approach, at each marching step the solution to Eqs. (6) is sought in an iterative manner. Let us denote the iteration level by the index n, so that the iteration at which we seek the solution is represented by the superscript $n+1$, and the previous iteration (known solution) is represented by the superscript n. If we assume that the solution at the $n+1$ level is close to the solution at the nth iteration, we can use a first-order Taylor series expansion around the previous iteration to write

$$\boldsymbol{f}_i^{j+1,n+1} \cong \boldsymbol{f}_i^{j+1,n} + \boldsymbol{A}_i^n \cdot \Delta \boldsymbol{q}^{n+1} \tag{17a}$$

$$\boldsymbol{s}^{j+1,n+1} \cong \boldsymbol{s}^{j+1,n} + \boldsymbol{M}^n \cdot \Delta \boldsymbol{q}^{n+1} \tag{17b}$$

$$\boldsymbol{h}^{j+1,n+1} \cong \boldsymbol{h}^{j+1,n} + \boldsymbol{A}_0^n \cdot \Delta \boldsymbol{q}^{n+1} \tag{17c}$$

where

$$\Delta \boldsymbol{q}^{n+1} = \boldsymbol{q}^{j+1,n+1} - \boldsymbol{q}^{j+1,n} \tag{18}$$

and the matrices A_0, A_i, and M are called the Jacobian matrices. It should be noted that in evaluating the Jacobian matrices and doing the Taylor series expansion around nth iteration, we only consider the flowfield variables as the unknowns and assume that we know the grid at the $j+1$ step.

Thus, we see that by expanding the solution around the nth iteration and using a two-point streamwise differencing, the governing PNS equations at the $n+1$ iteration can be written as

$$\begin{aligned}(A_1/\Delta\xi_1 - A_0)^n \cdot \Delta \boldsymbol{q}^{n+1} &+ [(A_2 - \varepsilon M)^n \cdot \Delta \boldsymbol{q}^{n+1}]_{,\xi_2} \\ &+ [A_3^n \cdot \Delta \boldsymbol{q}^{n+1}]_{,\xi_3} \\ &= -[\boldsymbol{f}_{i,\xi_i} - \varepsilon \boldsymbol{s}_{,\xi_2} - \boldsymbol{h}]^{j+1,n} = \boldsymbol{g}^{j+1,n}\end{aligned} \tag{19}$$

Differencing Scheme

Equation (19) is a vectorial equation involving derivatives in the ξ_1, ξ_2, and ξ_3 coordinate directions. Since the problem is elliptic in the ξ_2 and ξ_3 directions, we use central-differenced operators for all ξ_2 and ξ_3 derivatives, and a simple two-point backward-differenced approximation for the ξ_1 (streamwise) derivative. In the case of conventional noniterative PNS schemes,[1,2,4] it has been noted that a simple two-point backward differencing is nonconservative, as it results in truncation errors of order $O[(q^{j+1} - q^j)^2]$.

In contrast to the conventional noniterative PNS schemes, for the present scheme the two-point backward differencing formula involves truncation errors of roder $O[(\Delta \boldsymbol{q}^{n+1})^2]$. Thus, in the limit of convergence ($\Delta \boldsymbol{q}^{n+1} \to 0$), the two-point streamwise differencing of the present PNS scheme becomes conservative.

The fact that the present PNS scheme allows us to use a simple two-point backward differencing is not only important from a storage point of view, but it also gives the present scheme considerably improved capability for treating strong compression discontinuities. Experience with classical noniterative PNS schemes shows that these schemes encounter severe numerical difficulties when trying to step across regions of sudden geometry changes (such as strong compression corners). This is because across such discontinuities: 1) a simple second-order Taylor series expansion around the previous step is not accurate, and 2) the streamwise differencing of Eq. (19) is no longer conservative. For these reasons (and more), significant solution oscillations are observed when marching across a strong compression corner with a noniterative PNS scheme. In many cases the noniterative schemes just fail to step across such compression discontinuities. This problem seems to become more pronounced under high-altitude (low-density) re-entry conditions where even small body-slope changes have a strong impact on the solution.

Second-Order-Accurate Fully Implicit Smoothing

The governing equations [Eqs. (6)] are elliptic in the ξ_2 direction so that we used central-differenced approximations for all ξ_2 derivatives. However, as was also noted by Schiff and Steger,[1] the use of central-differenced schemes is typically associated with solution oscillations. This oscillatory behavior becomes more pronounced if the local velocities are small, so that the diagonal terms of the Jacobian matrices become relatively small. In order to damp these solution oscillations, it is necessary to add some additional higher-order diffusion terms to the governing PNS equations.

Recently we have developed a new second-order-accurate, fully implicit smoothing approach and have successfully applied it to the prediction of realistic re-entry flowfields. Using this approach, Eqs. (6) can be written as

$$\boldsymbol{f}_{i,\xi_i}^{j+1,n+1} = \varepsilon \boldsymbol{s}_{,\xi_2}^{j+1,n+1} + \boldsymbol{h}^{j+1,n+1} + \pi(\boldsymbol{q})(\Delta\xi_2)^2 \tag{20}$$

where the form of the vector $\boldsymbol{\pi}$ is chosen as

$$\begin{aligned}\pi(\boldsymbol{q}) = (1/4)\{&(A_1/\Delta\xi_1 - A_0) \cdot \Delta \boldsymbol{q}_{,\xi_2\xi_2} \\ &+ [(A_2 - \varepsilon M) \cdot \Delta \boldsymbol{q}_{,\xi_2\xi_2}]_{,\xi_2} + (A_3 \cdot \Delta \boldsymbol{q}_{,\xi_2\xi_2})_{,\xi_3}\}\end{aligned} \tag{21}$$

It has been shown in Ref. 10 that to second-order accuracy in $\Delta\xi_2$, we can rewrite Eq. (20) in terms of an intermediate solution $\boldsymbol{\chi}^{j+1}$ as

$$[\boldsymbol{f}_j(\chi^{j+1})]_{,\xi_j} = \varepsilon[\boldsymbol{s}(\chi^{j+1})]_{,\xi_2} + \boldsymbol{h}(\chi^{j+1}) + O(\Delta\xi_2)^2 \tag{22}$$

The actual solution that we seek at the $j+1$ step is related to this intermediate solution by the explicit transformation

$$\boldsymbol{q}^{j+1} = \boldsymbol{\chi}^{j+1} + \boldsymbol{\chi}_{,\xi_2\xi_2}\Delta\xi_2^2/4 \tag{23}$$

Pseudounsteady Formulation

As a result of the foregoing discussion, we see that by using a two-point backward differencing for the ξ_1 derivatives, central-difference approximations for the ξ_2 derivaties, including second-order-accurate smoothing, and expanding the solution around the nth iteration, the difference equations corresponding to the solution for the $n+1$ iteration at the $j+1$ marching step can be written in terms of an intermediate solution vector $\chi^{j+1,n+1}$ as

$$\begin{aligned}&[A_1/\Delta\xi_1 - A_0]^n_\ell \cdot \Delta\chi^{n+1}_\ell + [A^n_3 \cdot \Delta\chi^{n+1}]_{\ell,\xi_3}\\ &+ (1/2\Delta\xi_2)[A_2 - \varepsilon M]^n_{\ell+1} \cdot \Delta\chi^{n+1}_{\ell+1}\\ &- (1/2\Delta\xi_2)[A_2 - \varepsilon M]^n_{\ell-1} \cdot \Delta\chi^{n+1}_{\ell-1}\\ &= -[f_{i,\xi_i} - \varepsilon s_{,\xi_2} - h]^{j+1,n}_\ell \end{aligned} \tag{24}$$

We can simply write this block-pentadiagonal system as

$$\begin{aligned}&A^n \cdot \Delta\chi^{n+1}_{\ell-1} + B^n \cdot \Delta\chi^{n+1}_\ell + C^n \cdot \Delta\chi^{n+1}_{\ell+1}\\ &+ [A^n_3 \cdot \Delta\chi^{n+1}]_{,\ell,\xi_3} = g^{j+1,n}_\ell \end{aligned} \tag{25}$$

Thus, we see that as we converge to the final result, the right-hand side of Eq. (25) approaches zero. Under this limiting case, there is one unique solution to the above problem, which is $\Delta\chi = 0$ as long as B^n is a nonsingular matrix. Furthermore, we see that as long as B^n is nonsingular, the accuracy of the matrices A^n, B^n, and C^n really does not make any difference in the limit of convergence. Slight changes in these matrices would only change the nature of the pseudounsteady problem, which is of no concern to us except that it affects the convergence rates.

In the present approach, the crossflow derivatives on the implicit left-hand side are neglected, whereas these crossflow derivatives are retained to a second-order accuracy on the explicit right-hand side. This approach is in a way similar to the one followed by Lubard and Helliwell[5] and Agarwal and Rakich.[17,18] In the approaches of Refs. 5, 17, and 18, the so-called "backward" contributions of the implicit crossflow derivatives are retained while only the "forward" contributions are neglected. Our results showed that our approach and one similar to that of Lubard and Helliwell[5] resulted in almost identical solution response. However, the present approach not only represents a substantial saving in computer storage, but also a reasonable saving in computational time. By neglecting the implicit crossflow terms, the solution to the actual block-pentadiagonal system reduces to the solution of a set of line-by-line block tridiagonal equations. In our unpublished work, we have compared this line-by-line block-tridiagonal solution with a full block-pentadiagonal solution. These results showed that while the final converged solution was unaffected, the line-by-line solution took roughly one-sixth of the computing time (per iteration) required by the corresponding block-pentadiagonal solution.

The present scheme has the advantage that usually the A, B, and C matrices for the first iteration are a good approximation to A^n, B^n, C^n, respectively, at any subsequent iteration. Consequently, A, B, and C, are evaluated only once for the first iteration and kept unchanged for the remaining iterations. Thus, without affecting the final steady-state solution, we can write the block-pentadiagonal system of Eq. (24) as the following block-tridiagonal system:

$$A \cdot \Delta\chi^{n+1}_{\ell-1} + B \cdot \Delta\chi^{n+1}_\ell + C \cdot \Delta\chi^{n+1}_{\ell+1} = g^{j+1,n}_\ell \tag{26}$$

From Eq. (26) we see that we need to invert the matrices only once and store their inverted forms. Usually, the time taken for each iteration after the first iteration is only 10–15% of the time taken for the first iteration. After having obtained the intermediate solution $\chi^{j+1,n+1}$, the actual solution $q^{j+1,n+1}$ can be obtained very easily by using Eq. (23). This is an explicit expression for $\Delta q^{j+1,n+1}$ and can be processed with a negligible amount of additional computing time and effort.

Boundary Conditions

The problem represented by Eqs. (6) is a split-boundary-value problem, i.e., the equations are hyperbolic/parabolic in the ξ_1 direction and elliptic in the ξ_2 and ξ_3 directions. Thus, in order to solve the problem completely, we need initial conditions to be specified at the start of the marching procedure, and boundary conditions to be specified at the wall and at the outer bow shock.

Initial Conditions

For the present PNS scheme, the initial conditions to start the marching procedure are obtained from a viscous shock-layer (VSL) blunt-body solution. The quality of such VSL solutions has been discussed in great detail in Refs. 19–22. For the present scheme, the VSL blunt-body solution was interpolated to obtain the starting solution at the initial data plane (IDP) for the PNS solution. Typically, we choose this starting location to be approximately 2–3 nose radii downstream of the stagnation point.

Wall Boundary Conditions

For the present PNS scheme, the boundary conditions at the wall consist of six independent relations representing the nature of the gas and the physical conditions at the wall. These conditions are 1) the equation of state for a perfect gas ($\gamma p = \rho T$), 2–4) the no-slip condition for u_i velocity component ($\rho u_i = 0$), 5) specified wall temperature ($\rho = \gamma p/T_w$), and 6) zero pressure derivative in the ξ_2 direction ($p_{,\xi_2} = 0$).

Outer Boundary Conditions

The boundary conditions at the outer bow shock are more involved than the boundary conditions at the wall. In the present approach, we have used a shock-propagation scheme very similar to the one used by Chaussee et al.[4] This scheme uses a combination of inviscid compressible flow equations to predict the $p_{,\xi_1}$ value behind the shock at the jth marching station. Using this pressure derivative and a simple Euler integration, the pressure behind the shock at the $j+1$ marching step is predicted, and the remaining conditions behind the shock are determined from the Rankine-Hugoniot shock-crossing relations. The shock location at the $j+1$ step is then predicted using the conditions behind the shock at the $j+1$ step, and the fact that the shock is a $\xi_2 = \text{const}$ and $\xi_3 = \text{const}$ surface.

Streamwise Stability Considerations

Controlling the Time-Like Character of PNS Equations

In order to eliminate the problem of departure, various researchers have devised different approaches. Most notable of these are the sublayer approximation used by Schiff and Steger,[1] the approach of Vigneron et al.,[3] and the step-size control methods of Lubard and Helliwell[5] and Lin and Rubin.[9] The departure analyses of Schiff and Steger[1] and Vigneron et al.[3] were based on looking at the character of the governing equations and the corresponding eigenvalues of the system in the viscous and inviscid limits. In the case of the Schiff-Steger approach,[1] the sublayer pressure was assumed to be known and a constant. By doing so, it was observed that the resulting governing equations became time-like even in the sublayer region, and the problem of departure was thus resolved. Vigneron et al.,[3] on the other hand, assumed that in the subsonic sublayer region only a fraction ω of the streamwise pressure derivative ($p_{,\xi_1}$) was to be included. With such an assumption, the governing equations also became time-like in the subsonic sublayer region, as long as $\omega \le \gamma M^2/[1+(\gamma-1)M^2]$.

In the case of Lubard and Helliwell,[5] it was observed that if the streamwise pressure derivative ($p_{,\xi_1}$) was estimated explicitly, convergent (stable in the iteration direction) and departure-free (streamwise stable) marching solutions were possible if $\Delta x_{min} < \Delta x < \Delta x_{max}$, where the lower bound was the constraint for a departure-free solution, and the upper bound was the constraint for the iteration convergence. The approach of Lin and Rubin[9] for suppressing departure solutions resembles that of Lubard and Helliwell[5] in suggesting a minimum step-size constraint when the $p_{,\xi_1}$ term is backward differenced. They also suggested that if $p_{,\xi_1}$ was forward differenced, departure could be prevented entirely. Such a forward-differenced approach requires the use of a global relaxation procedure (global iterations). However, the present study concentrates on "single-sweep" PNS procedures only, and not on globally iterated PNS methods.

In the aforementioned single-sweep PNS schemes, we can see that the possibility of departure solutions (streamwise instabilities) exists, and the constraints developed to prevent departure involve a special treatment of either the sublayer pressure or of its streamwise derivative. If we look into these schemes for preventing solution departure, and consider the mathematical reasonings leading to their development,[1,3,5] we see that (in a way) they presuppose that the streamwise pressure derivative is the culprit, and something must be done with respect to it. In each of these cases, the reasoning is based on the original suggestion by Rubin and Lin[8] of an "elliptic pressure effect" in the streamwise direction. This suggestion by Rubin and Lin was based on a consideration of interacting boundary-layer theory and, also, assumed an incompressible flow (Ref. 8, p. 24). At least in the case of Vigneron et al.[3] and Lubard and Helliwell,[5] this presupposition about the treatment of the $p_{,\xi_1}$ term is obvious. Consider the case of Vigneron et al.[3] and their eigenvalue analysis. It should be noted that in this analysis the $p_{,\xi_1}$ term is not the only factor contributing to the eigenvalues of the differential system. Suppose we choose to retain a fraction ω of the normal pressure derivative, rather than the streamwise pressure derivative. In this case, an analysis similar to the one done by Vigneron et al. shows that for stability we require $\omega = 1$ for $M > 1$ and $\omega \le M_x^2 M_y^2/4(1 - M_x^2)$ for $M < 1$. the validity of this approach is clearly demonstrated if we recognize that a more restrictive case ($\omega = 1$ for $M > 1$ and $\omega = 0$ for $M < 1$) is conceptually the same as the Schiff-Steger sublayer approximation.

Similarly, in the work of Lubard and Helliwell (Ref. 5, pp. 55–61), it was seen that if the $p_{,\xi_1}$ term in the u-momentum equation and the energy equation were evaluated explicitly, certain contributions to the amplification matrix were removed and the corresponding eigenvalues were ≤ 1 as long as $\Delta x_{min} < \Delta x < \Delta x_{max}$. The approach of Schiff and Steger[1] to prevent departure is different only in the sense that it specifies the sublayer pressure directly rather than specifying its streamwise derivative ($p_{,\xi_1}$). Nevertheless, even in this case it is done so because certain problematic contributions to the eigenvalues of the differential system are removed, and the eigenvalues become always real. This in turn implies that the governing differential equations become always time-like.

Thus, as can be seen from the above discussion, in all of the earlier single-sweep PNS treatments, the problem of departure (streamwise instability) is removed by actually performing mathematical manipulations (eliminating or changing certain terms) such that either the eigenvalues of the differential system[1,3] or the eigenvalues of the amplification matrix[5] have acceptable values. Now we pose a question: Is it possible to do a mathematical manipulation of the governing PNS equations such that an acceptable time-like character of the governing equations can be obtained without having to neglect certain terms either in part or completely? This question is the focal point of the new PNS scheme being proposed. In this new PNS scheme, the following idea is presented: If we consider pressure as an additional unknown quantity and solve the algebraic equation of state simultaneously (in a coupled manner) with the differential equations governing the conservation of mass, momentum, and energy, it is possible to have an appropriate time-like marching scheme without having to specify either a constant sublayer pressure (Schiff-Steger approach), or to neglect a part of the streamwise pressure derivative (approach of Vigneron et al.), or to estimate the streamwise pressure derivative explicitly (Lubard-Helliwell approach).

The conceptual development of the proposed PNS scheme uses the formulation of Schiff and Steger[1] as the starting point. In order to eliminate this problem of complex eigenvalues in the sublayer region, Schiff and Steger assumed that the pressure in the sublayer is specified as a constant, and "... is not a function of $\boldsymbol{q}$ (the local vector of unknowns) ... ." (Ref. 1, p. 4). In doing so, it is indirectly assumed that the sublayer pressure p_s is an additional unknown quantity. However, one is faced with the problem of how to go from basically five unknowns (ρ, ρu, ρv, ρw, and e) in the supersonic region to six unknowns in the subsonic region (ρ, ρu, ρv, ρw, e, and p_s). This problem was resolved through an approximation in which the value of p_s was estimated by extrapolation from the previous steps. It becomes clear from this discussion that the sublayer approximation is at the expense of satisfying the equation of state in the sublayer region. Once this sublayer solution has been obtained, the updated sublayer pressures are recomputed from this solution using the equation of state. However, this application of the equation of state occurs after the solution has already been obtained, so that although the derived pressures may satisfy the equation of state, the solution of the remaining unknown variables (ρ, ρu, ρv, ρw, and e) may not accurately satisfy the governing conservation equations.

In the present approach, we conceptually extend the Schiff-Steger ideas, and rather than just considering pressure as an additional unknown in the subsonic sublayer region, we consider it as an additional unknown all across the shock layer. That is to say, we retain the pressure terms in the conservation equations the way they are (i.e., we do not replace them with appropriate differentiated forms of the equation of state), and we include the algebraic equation of state as an additional equation to be solved simultaneously. From a simplified version of these PNS equations, we see that in doing so we are able to remove the contribution of the speed of sound from the eigenvalues of the system in much the same way as the sublayer approximation does, but without having to impose the constant sublayer pressure condition.

Results of Simplified Analysis for Departure

In order to simplify the required mathematics, let us restrict ourselves to 1) two-dimensional flows, and 2) an evenly spaced square grid such that $\xi_{1,x} = \xi_{2,z} = 1$ and $\xi_{1,z} = \xi_{2,x} = 0$. Furthermore, we choose to approximate the equation of state for a perfect gas by

$$\gamma p - \rho T + \theta(p_{,\xi_1} + p_{,\xi_2}) = 0 \tag{27}$$

where the coefficient θ is chosen such that $\theta \cong 0$ and for all practical purposes. It should be noted that the use of this coefficient θ is for the sole purpose of the following stability analysis, and not for the actual solution scheme. In other words, the actual solution corresponds to the use of $\theta = 0$. It should be noted that the use of θ in Eq. (27) corresponds to the use of the θ term in formulation II of the model problem, where it was demonstrated that the choice $\theta \to 0^+$ did not produce any singularity in the final solution. In Ref. 23 we have used Eq. (27) along with the governing PNS equations to demonstrate numerically that small (10^{-6}–10^{-24}) values of θ can be used without any adverse or singular behavior. Some sample results for these tests are shown in Fig. 2. This figure shows the wall-pressure distributions for the flow over a 7 deg sphere cone at a flight altitude of 80 kft and a flight Mach number of 25.

With the equation of state given by Eq. (27), and after neglecting the viscous terms containing the contributions of w and $w_{,\xi_3}$, we can write the simplified PNS equations as

$$(A_1^n \cdot d)_{,\xi_1} + (A_2^n \cdot d)_{,\xi_2} - \varepsilon[M^n \cdot d]_{,\xi_2} - A_0^n \cdot d$$
$$= -[f_{1,\xi_1} + f_{2,\xi_2} - \varepsilon s_{,\xi_2} - h]^n \tag{28}$$

where $d = \Delta q^{n+1}$ and A_1^n, A_2^n, and M^n are the Jacobian matrices. If we assume that A_1^n, A_2^n, and M^n do not change with ξ_1 and ξ_2 (a frozen coefficient analysis), we can write

$$A_1^f \cdot d_{,\xi_1} + A_2^f \cdot d_{,\xi_2} - B^f \cdot d_{,\xi_2\xi_2} + c = 0 \tag{29}$$

Although the preceding equation is a significantly simplified version of the original PNS equations, it is still difficult to study directly. As a further simplification, we choose to look separately at the viscous and inviscid limits of Eq. (29).

Inviscid Limit

The inviscid limit of Eq. (29) can be written as

$$d_{,\xi_1} + N_1 \cdot d_{,\xi_2} + c_1 = 0 \tag{30}$$

This equation is now in a form that can be easily studied. For Eq. (30) to be stable, the ξ_1 direction should be a valid marching direction. In other words, Eq. (30) has to be always hyperbolic/parabolic. This condition is satisfied if the eigenvalues of N_1 are all real. If for simplicity we assume that $w < < u$, then an eigenvalue analysis gives the eigenvalues of N_1 as $\lambda_i = (1, w/u, w/u, w/u, w/u)$. Thus, we see that all of the eigenvalues of N_1 are unconditionally real. That is to say, the simplified PNS equations being studied are unconditionally marching-like in the inviscid limit and represent a stable marching scheme in the subsonic and as the supersonic flow region.

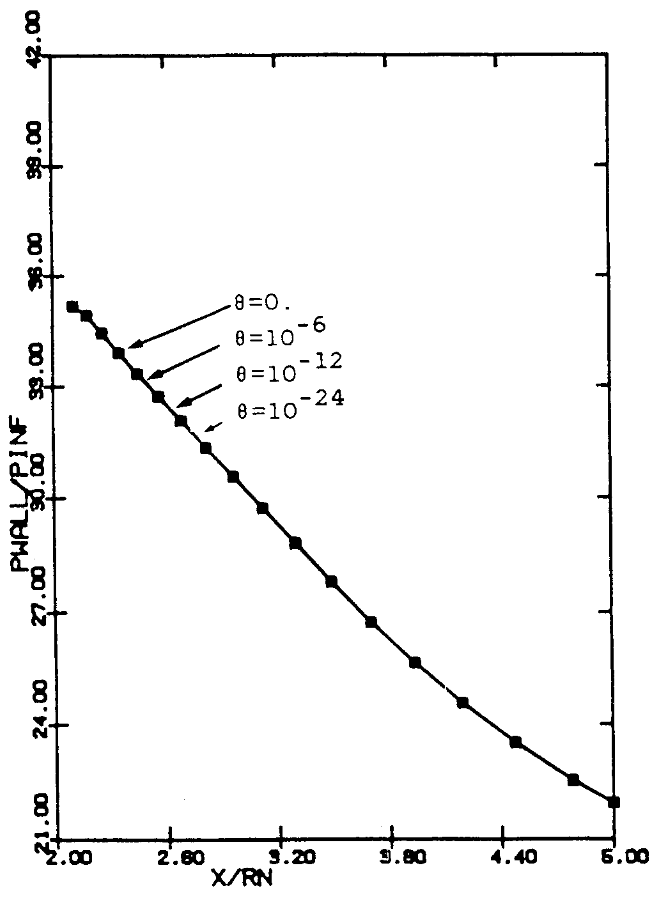

Fig. 2 Effects of θ on wall pressures.

Viscous Limit

In the viscous limit, Eq. (30) simplifies to

$$d_{,\xi_1} = N_2 \cdot d_{,\xi_2\xi_2} + c_2 \tag{31}$$

In this form, the stability analysis becomes much simpler. The criterion of a stable marching scheme requires that Eq. (31) should be parabolic. The parabolic character depends on the eigenvalues of N_2, which should be real. Furthermore, in order to have positive diffusion effects in the ξ_1 direction, these eigenvalues should also be positive. Thus, for the viscous limiting case to be stable, the eigenvalues of N_2 should be real and positive. An eigenvalue analysis of N_2 shows that the eigenvalues are $\sigma_i = (0, 0, \varepsilon\mu/Pr\rho u, 4\varepsilon\mu/3\rho u, \varepsilon\mu/\rho u)$. Thus, the eigenvalues σ_i are always real; however, they are positive only if $u > 0$. That is to say, as long as no flow reversal occurs in the streamwise direction, the viscous limit of the simplified PNS equations is also unconditionally marching-like. Since flow reversal means axial separation, this streamwise stability requirement actually tells us that a single-sweep solution of these PNS equations cannot be marched through regions of axial flow separation. Of course, this conclusion comes as no surprise and has been a well-accepted fact in fluid mechanics for a long time.

Results and Discussion

In order to test the various accuracy, stability, and efficiency claims of the proposed three-dimensional PNS scheme (PNSPG3D), we studied the laminar flow around a 7 deg sphere-cone vehicle at a flight altitude of 80 kft and an angle of attack of 5 deg. The freestream Mach number for this case was 25, and the wall temperature was kept constant at 2000°R. The sphere-cone vehicle is 30 nose radii long and has a nose radius of 0.041667 ft. The freestream conditions are given in Table 1.

The results have been compared with the corresponding predictions of the HYTAC PNS code of Helliwell et al.[6] Comparisons have also been made with the corresponding inviscid predictions, and the effects of grid refinement have been studied. The following sections discuss these results in detail.

Code Comparison and Solution Accuracy

Figures 3–5 show some of the results for the 5 deg angle-of-attack test case being considered. These results have been compared with the results of the HYTAC PNS code and the inviscid NOL3D code.[24] It should be pointed out that both HYTAC and PNSPG3D solutions use 50 body-normal or axis-normal grid points and 9 circumferential grid points. The grid stretching used in the two schemes is, however, different. The PNSPG3D grid is finer near the wall. The grid spacing near the wall for the PNSPG3D solution is 5×10^{-3}% of the local shock-standoff distance, whereas for the HYTAC PNS code it is 3.82×10^{-2}%.

The wall-pressure distributions for this case show that the two viscous results are in very good agreement with each

Table 1 Freestream conditions

Altitude, kft	80.000
Mach number	25.000
Reynolds number	
(based on nose radius)	2.92E + 5
Pressure, lb/ft^2	58.562
Density, slug/ft^3	8.64E − 5
Temperature, °R	395.067
Velocity, ft/s	2.44E + 4
α, deg	5.000

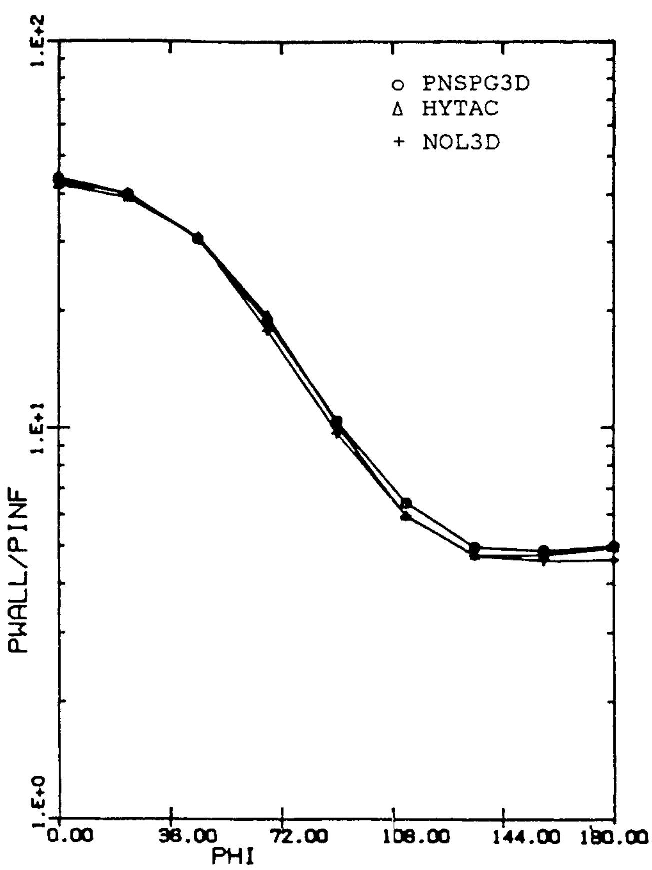

Fig. 3 Crossflow distribution of wall pressures at the body end.

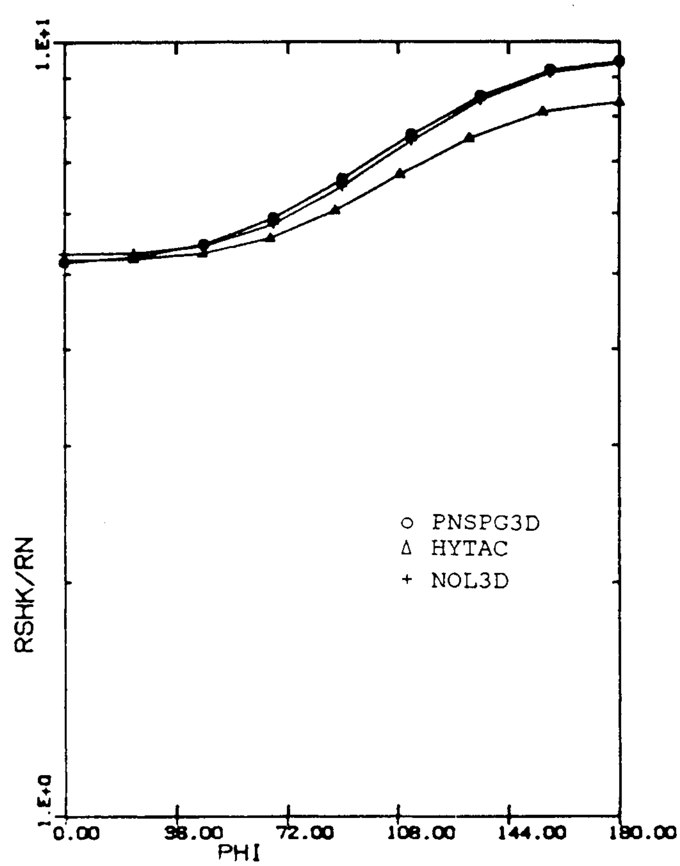

Fig. 4 Crossflow variation of shock-standoff distances at the body end.

other and with the inviscid predictions in the afterbody region. There appear to be small differences in the viscous and inviscid results in the forebody region ($3 < x/Rn < 10$), which are a result of the viscous blunt-nose effects. The crossflow distributions of wall pressure at the body end are compared in Fig. 3, while Fig. 4 shows the crossflow variations of the shock-standoff distance. These results show that the PNSPG3D predictions are in very good agreement with the inviscid predictions, whereas the HYTAC scheme predicts a much thinner shock layer on the leeward side.

Figure 5 shows that the agreement between heat-transfer predictions of the two PNS codes worsens as we move from the windward to the leeward side. On the windward side HYTAC overpredicts the heat transfer by 20–25%, whereas on the leeward side this overprediction is of the order of 40–50%. However, our grid refinement studies clearly show that these differences are solely due to the coarseness of the HYTAC grid near the wall. There are differences in the predicted skin-friction distributions; however, they also show similar trends.

Effects of Grid Coarseness on Solution Accuracy

The results presented earlier (see Figs. 3–5) show some significant differences between the wall heat-transfer and skin-friction predictions of the present PNSPG3D calculations and the HYTAC PNS results. In order to study the reasons behind these differences, we recognized that the two PNS schemes used different grid stretching near the wall (the PNSPG3D results used a finer grid). For this purpose we first tried to run the HYTAC PNS calculations with the same grid stretching used in the PNSPG3D calculations. The use of this fine grid caused convergence problems in the HYTAC PNS scheme. Repeated attempts with various marching step sizes failed to resolve these problems. Thus, we took the other alternative of running the present PNSPG3D calculations

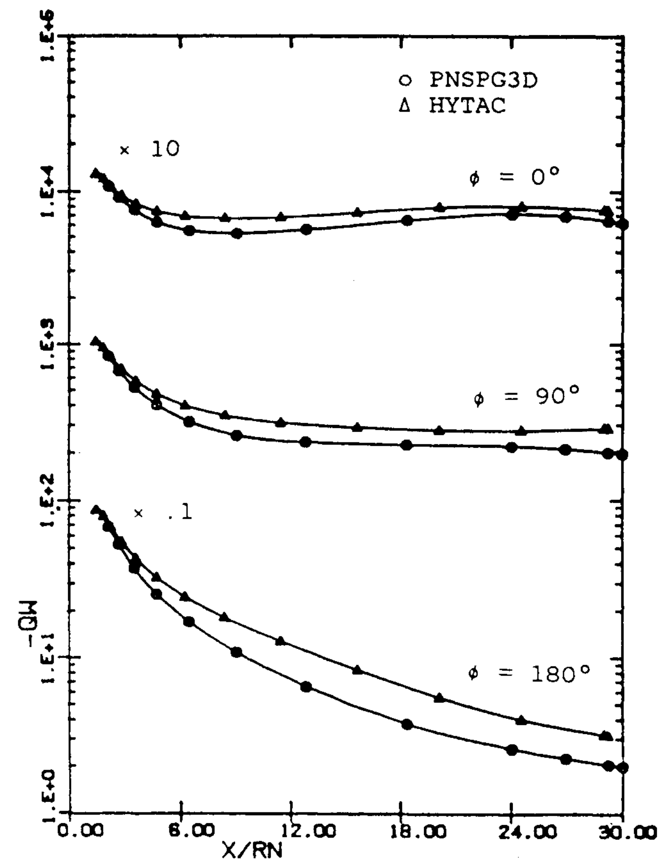

Fig. 5 Axial distribution of wall heat-transfer rates.

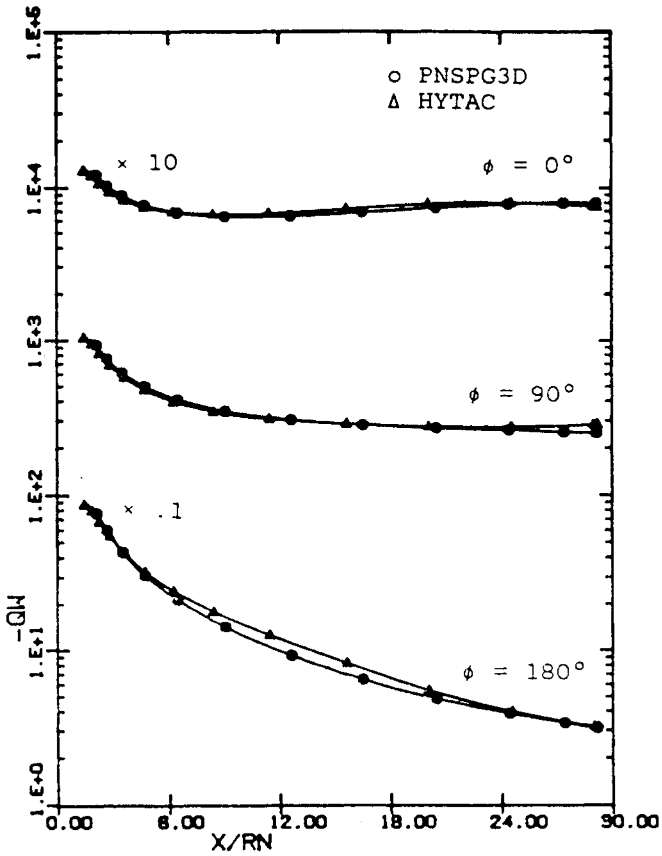

Fig. 6 Effects of grid coarseness on the wall heat-transfer rates.

Table 2 Comparison of total computing times[a]

Code	X/RN from–to	Grid $\xi_1 \times \xi_2 \times \xi_3$	t, m:s	τ, s
HYTAC	2.–30.	38 × 50 × 9	3:30	0.01228
PNSPG3D	2.–30.	43 × 50 × 9	2:32	0.00786

[a]On IBM 3084 with H-compiler and OPT = 2 optimization.

with the same coarse grid as the one used in the HYTAC PNS calculations. The wall-pressure distributions for these calculations are the least affected by grid refinement (or, equivalently, grid coarseness). The heat-transfer predictions for these calculations are shown in Fig. 6. This figure shows that with the same grid stretching the two PNS predictions are in very good agreement with each other. The worst agreement is on the leeward side, and even those differences are within 10–15% of each other, with the HYTAC PNS results being on the overpredictive side.

Computing Times

One of the important results of the present study was the computing time consideration. Table 2 shows the computing times for the present calculations and shows that in terms of the overall computing times, the present PNSPG3D scheme represents an improvement of 36% over the HYTAC PNS computing times. However, in terms of the computing time per grid point, the present PNSPG3D computing times represent an improvement of 56% over the corresponding HYTAC PNS calculations. This difference on a per-grid-point basis is smaller primarily because the PNSPG3D calculations used a larger number of streamwise marching steps (43 as compared to 38 for HYTAC).

Conclusions

A new three-dimensional perfect-gas scheme based on the thin-layer parabolized Navier-Stokes (PNS) equations has been developed. The mathematical and conceptual background leading to this PNS scheme has been discussed in detail. This PNS scheme was used to study the three-dimensional flowfield around a 7 deg sphere-cone re-entry vehicle at a 5 deg angle of attack. The mathematical and numerical results of the present study lead to the following conclusions:

1) The results of a detailed analysis of a model differential/algebraic problem show that the conditional or unconditional time-like character of such differential/algebraic systems depends on the way the problem is formulated.

2) Based on the background provided by the model problem, we have seen that if we treat the pressure as an additional unknown and solve for it in a coupled manner using the algebraic equation of state for a perfect gas, we obtain an unconditionally time-like problem.

3) It is shown that the present approach can be viewed as an extension of the Schiff-Steger sublayer approximation. A simplified eigenvalue analysis for possible departure behavior is presented, and it shows that the present scheme indeed represents an unconditionally time-like character in the inviscid and the viscous limiting cases.

4) A new second-order-accurate smoothing approach has been successfully used to damp possible solution oscillations, and a pseudounsteady solution approach has been used to improve the solution efficiency drastically. This pseudounsteady solution approach merely simplifies the solution without affecting the final solution accuracy.

5) The three-dimensional moderate angle-of-attack (5 deg) flow around a 7 deg sphere-cone test vehicle is studied, and comparisons with the HYTAC PNS calculations show that the present PNS scheme is more accurate and permits finer grids to be used near the wall.

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